

Purely quantum memory in closed systems observed via imperfect measurements

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The detection and quantification of non-Markovianity, a.k.a. memory, in quantum systems is a central problem in the theory of open quantum systems. There memory is a result of the interaction between the system and its environment. Little is known, however, about memory effects induced by imperfect measurements on closed systems, where an entanglement with the environment is not possible. We investigate the emergence and characteristics of memory in closed systems observed via imperfect stroboscopic quantum measurements yielding coarse-grained outcomes. We consider ideal and two kinds of imperfect measurements: von Neumann measurements—the analogue of classical lumping—which destroy any coherence in the system, and *genuinely quantum-lumping* Lüders measurements preserving certain quantum correlations. Whereas the conditions for Markov dynamics under von Neumann lumping are the same as for classical dynamics, quantum-lumping requires stronger conditions, i.e. the absence of any detectable coherence. We introduce the concept of *purely quantum memory* having no classical counterpart. We illustrate our results with a quantum walk on a lattice and discuss their implications for dissipative dynamics and decoherence effects induced by more realistic measurements.

1 Introduction

In 1932 John von Neumann established that upon a quantum measurement, the quantum state of

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a system is projected (or collapsed) onto a one-dimension subspace of the Hilbert space associated with the measurement outcome [1]. In stark contrast to classical measurements, quantum measurements have drastic effects on the system's evolution because they destroy the quantum coherence, giving rise, e.g. to the Zeno effect [2] and quantum localization [3]. In the case when quantum measurements are performed stroboscopically and one may also consistently define a first-detection time (as the quantum analog of the first-passage time) [4–7], the destruction of coherence has been associated with the “classicality” and the Markov property of the quantum evolution [8–10].

The situation becomes much more intriguing if measurement outcomes are degenerate; according to von Neumann the collapsed state should then correspond to a *statistical mixture* of one-dimensional projections. However, in 1950 Gerhart Lüders noted that these projections are (i) *not* uniquely defined for a degenerate measurement, and (ii) a degenerate measurement preserves some quantum coherence [11, 12]. Nowadays the Lüders concept of measurement under ideal conditions [13] is fully accepted, but the von Neumann postulate still remains useful in several circumstances [14–17] where the measurement is affected by noise.

We look at this old question from a conceptually different angle, i.e. from the point of view of a non-Markovian evolution on a *lumped* (coarse-grained) quantum state space (see [18–31] for classical examples). Namely, both von Neumann's and Lüder's measurements *lump* indistinguishable individual outcomes onto distinguishable sets called *mesostates*. The dynamics projected onto mesostates preserves memory of the past evolution, inducing non-Markovian statistics [18, 19, 32, 33]. In the classical setting memory is known to emerge when statistically distinct

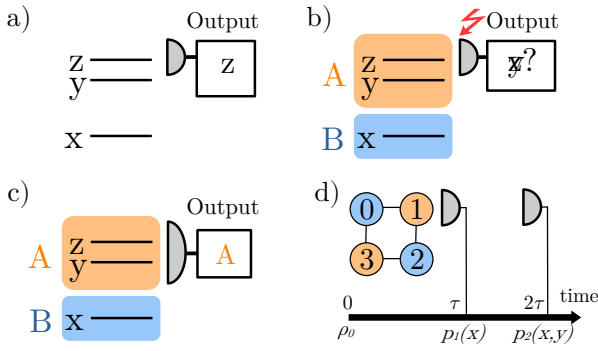


Figure 1: a) Sketch of an ideal quantum measurements: The measurement apparatus distinguishes between all individual quantum states. b) von Neumann measurement: A quantum state is measured but the noise makes the outcome indistinguishable amongst groups of certain states. The latter become *lumped* into distinguishable sets called *mesostates* $A = \{z, y\}$, $B = \{x\}$..., which are clearly differentiated by the measurement apparatus. c) Lüders measurement of a single mesostate: The measurement apparatus is intrinsically unable to distinguish sets of states, such that the output are directly lumped mesostates. d) Stroboscopic measurement of a quantum system. The system shown is a quantum lattice with 4 sites and two mesostates (blue-orange).

microscopic transitions become projected onto mesostate-transitions as a “state-space contraction” [18, 19, 27–29, 29–33]. The detection, quantification, and understanding of quantum non-Markovian evolutions is even more subtle and is a central problem in open quantum systems theory [34–38] inspiring numerous dedicated experiments [39–43]. In open quantum systems, memory is well known to be a result of the interaction between the system and its environment [34–37, 44–49]. However, since the quantum measurement process directly affects the system’s dynamics, different measurement protocols imply starkly different memory effects [50].

In this work, we describe how Lüders and von-Neumann type coarse-grained measurements give rise to different kinds of memory. In particular, we find that the von-Neumann measurement exhibits memory effects similar to a classical non-Markovian process, while the Lüders measurement gives rise to purely quantum features. The Lüders measurement requires stronger conditions to recover Markovian behavior.

The article is organized as follows: in Section 2 we define the effects of ideal von-Neumann

and Lüders measurements on a quantum system. In Sec. 3 we describe the evolution of quantum systems under general *consecutive* measurements, and discuss how the different measurement schemes imply non-Markovian lumped statistics. To conclude, in Sec. 4 we present an example of a real system where von-Neumann measurements give rise to fully Markov statistics, whereas Lüders measurements cause purely quantum memory effects.

2 Imperfect measurements

A prospective quantum measurement that can not determine completely the state of the system, is called an *imperfect measurement*; otherwise, the measurement is *ideal*. The typical origin of imperfect measurements is noise affecting the detector, inherently disabling an accurate record of measurement outputs of a projective measurement [51, 52]. Other examples include weak measurements [5, 53–55], non-demolishing [56–60], hidden qubits [61], or indirect measurements [6, 62]. In these examples, the information about the actual state of the system is coarse-grained (or *lumped*) into a set of fewer measurement outcomes called *mesostates*. We observe that in such a noisy measurement, the resulting quantum state is fully collapsed, i.e. the resulting state is a classical mixture [52]. However, in certain measurements, such as the weak measurement, the resulting state does *not* fully collapse certain subspaces of the system. Following the discussion in the Introduction, we call *von-Neumann measurements* those that fully collapse the state of the system during the process, and *Lüders measurements* those which preserve certain quantum coherences.

In this section, we state the models of *ideal* von-Neumann and Lüders measurements, respectively, depicted in Fig. 1a-c. Let the possible measurement outcomes be x, y, z , e.g. the energy levels of the hydrogen atom, or the sites of a quantum lattice. The ideal measurement (Fig. 1a) distinguishes between individual outcomes; the probability of obtaining each outcome x is given by the Born rule $p(x) = \text{Tr}\{\Pi_x \rho_0\}$, where $\Pi_x = |x\rangle\langle x|$ is a rank-1 projector associated with the outcome x and ρ_0 the density matrix representing the initial preparation of the quantum system. After the measurement, the ini-

tial state is collapsed onto the corresponding outcome subspace $\rho_0 \rightarrow \Pi_x \rho_0 \Pi_x / p(x)$.

Now consider the same measurement but performed by a noisy apparatus, see Fig. 1b. Here the probability of each outcome is still given by the Born rule. However, the thermal or electric noise renders the exact readout impossible and hence produces blurred outcomes $z, y, \dots$, such that we can only resolve mesostates $X = \{z, y, \dots\}$ these blurred outcomes belong (i.e. are *classically lumped*) to. The probability of obtaining an outcome inside mesostate $X = A$ is $p^C(A) = p(z) + p(y) + \dots = \sum_{x \in A} p(x)$ (analogously for every mesostate). The quantum state of the system is fully collapsed, such that the resulting state after the measurement is the weighted sum of all the possible outcomes in A , $\rho_0 \rightarrow \rho_A^C = \sum_{x \in A} \Pi_x \rho_0 \Pi_x / p^C(A)$. This corresponds to the von Neumann measurement.

In the third setting (see Fig. 1c) the apparatus is unable to distinguish z, y even in the absence of noise. Here, the measurement outcomes are directly the mesostates $X_i = A, B, \dots$. The probability of observing the mesostate A is $p^Q(A) = \text{Tr}\{\bar{\Pi}_A \rho_0 \bar{\Pi}_A\}$ with the high-rank projector $\bar{\Pi}_A \equiv \Pi_z + \Pi_y + \dots = \sum_{x \in A} \Pi_x$. After the measurement the quantum state is projected according to Π_A , $\rho_0 \rightarrow \rho_A^Q = \bar{\Pi}_A \rho_0 \bar{\Pi}_A / p^Q(A)$, corresponding to the Lüders measurement. Notably, the resulting ρ_A^Q is a block diagonal matrix; the non-diagonal elements represent the quantum coherence surviving the Lüders measurement, preserving information about the previous state of the system. We will show that this *purely quantum* memory effect prevents Markovian observed evolutions. To distinguish the two, we will in the following refer to the statistics $p^C(A)$ as *classical lumping* and to $p^Q(A)$ as *quantum lumping*.

We remark that the outcomes of the Lüders and von-Neumann measurements are identical, they only differ in the resulting quantum state. A direct method to distinguish these measurements corresponds to performing quantum tomography of the state after measurement [16]. However, one can also differentiate between these measurement types by performing consecutive measurements. For example, in [5, 53, 54] the authors consider the role of surviving coherence along repeated Lüders measurements on the statistics of the first detection-time.

3 Quantum system under consecutive measurements

Following the previous discussion we now consider the scenario of consecutive measurements on a quantum system. In this scenario, the quantum system is instantaneously measured at discrete times $t_1, t_2, \dots$. Between any two consecutive measurements, the system's evolution is given by an arbitrary trace-preserving map $\Lambda(t_1, t_2)$, $\rho(t_2) = \Lambda(t_1, t_2) \circ \rho(t_1)$. For example, if the system is isolated the evolution is given by the *unitary* map $U(t_2 - t_1) = \exp[-iH(t_2 - t_1)/\hbar]$, $\rho(t_2) = \mathcal{U}(t_2 - t_1) \circ \rho(t_1) = U\rho(t_1)U^\dagger$, where H is the Hamiltonian of the system.

The joint probability distribution of the outcomes of $n + 1$ consecutive measurements is [2]

$$p[\{o_i, t_i\}_{i \in [0, n]}] = \text{Tr}\{P_{x_n} \circ \Lambda(t_n, t_{n-1}) \circ P_{x_{n-1}} \dots \circ \rho_0\}, \quad (1)$$

were $P_{x_n} \equiv \Pi_{x_n}$ if the measurements are ideal with outcomes $o_i \equiv x_i$, and $P_n \equiv \bar{\Pi}_{X_n}$ if the measurements are of the Lüders type with $o_i \equiv X_i$. As we show below, the statistics of von-Neumann outcomes is obtained by lumping statistics of ideal measurements.

The memory effects in Eq. (1) can manifest in several ways. In the conventional picture, the quantum system is in contact with a certain environment, able to retain some information about the past evolution of the system [34–36, 63–65]. Due to this memory, the maps $\Lambda(t_n, t_{n-1})$ could depend on previous measurement outcomes. Within this framework, the non-Markovianity of the system is usually identified with the divisibility of the maps $\Lambda(t_n, t_{n-1})$ into completely-positive intermediate maps [35]. However, this approach can not capture the memory effects introduced by different measurement instruments acting on the system [66]. There exist different approaches to Markovianity trying to reconcile the memory effects of the environment with the consecutive-operations framework [44–47]. The common point in any consistent definition of quantum Markovianity is recovering the classical definition in the classical limit, this is, that Eq. (1) yields

$$p[x_n, t_n | x_{n-1}, t_{n-1}; x_{n-2}, t_{n-2}; \dots] = p[x_n, t_n | x_{n-1}, t_{n-1}]. \quad (2)$$

In any other case the process is non-Markovian. From this one can conclude that there is a unique way of “Markovian”, but there are multiple fundamental ways of “non-Markovian”. A natural distinction amounts to identifying different environments with *classical memory*, and *quantum memory* (or entanglement) [48, 49]. In the spirit of this distinction, we here investigate how memory effects depend on the measurement instruments, using the paradigmatic Lüders (preserving quantum memory) and von-Neumann (preserving *no* quantum memory) measurements as representative types of instruments.

We conclude this discussion with a relevant observation. The repeated measurement of a Markovian stochastic process can recover non-Markovian statistics [67–69], even if the measurement is not affecting the dynamics [27]. This happens because of the loss of information due to the coarse-graining underlying the measurements or *lumping* [28–31]. To focus on this phenomenon, we will in the following assume that the dynamics between quantum measurements is fully Markovian, i.e. no memory can be stored in the environment. We consider that the measurement statistics are Markovian when they satisfy Eq. (2) given a fixed measurement instrument [9, 10, 40].

We now consider Eq. (1) explicitly for ideal von-Neumann and Lüders measurements, respectively, and discuss their connection to Eq. (2). For simplicity we restrict ourselves to the scenario of *stroboscopic measurements* (see Fig. 1d). In this scenario the measurements are taken at fixed equidistant times $\tau_n = n\tau$ for $n \in \mathbb{N}_0$. Since the environment has no memory, we assume that $\Lambda(t_2, t_1)$ is a completely positive trace-preserving map depending on τ exclusively, $\Lambda(t_2, t_1) \equiv \Lambda(\tau = t_2 - t_1) = \Lambda_\tau$. After $n + 1$ ideal consecutive measurements we obtain the joint probability distribution of the outcomes [2]

$$p_n[\{x_i, \tau_i\}_{i \in [0, n]}] = \text{Tr}\{\Pi_{x_n} \circ \Lambda_\tau \circ \Pi_{x_{n-1}} \circ \Lambda_\tau \circ \dots \circ \Pi_{x_0} \circ \rho_0\}, \quad (3)$$

where $\Pi_x \circ \rho \equiv \Pi_x \rho \Pi_x$, and $\Lambda_\tau \circ \rho \equiv \Lambda_\tau \rho \Lambda_\tau^\dagger$.

Interestingly, the distribution in Eq. (3) in general does *not* satisfy the Kolmogorov condition $p_{n-1}[\{x_i, \tau_i\}_{i \in [0, n] \setminus k}] = \sum_{x_k} p_n[\{x_i, \tau_i\}_{i \in [0, n]}]$ for some $k \in [0, n]$, because the measurement destroys coherence [36] and some multi-time correlations may be ill defined [9]. Nevertheless, we can always consider

the conditional probability of consecutive *ideal* measurements $p[x, \tau_n | \{x_i, \tau_i\}_{i \in [0, n-1]}] \equiv p_n[\{x_i, \tau_i\}_{i \in [0, n]}] / p_{n-1}[\{x_i, \tau_i\}_{i \in [0, n-1]}]$ [9] and using Eq. (3) we obtain

$$p[x_n, \tau_n | \{x_i, \tau_i\}_{i \in [0, n-1]}] = p[x_n, \tau_n | x_{n-1}, \tau_{n-1}] \equiv T_{x_n}^{x_{n-1}}, \quad (4)$$

where $T_x^y = |\langle x | \Lambda_\tau | y \rangle|^2$ is the transition matrix, whose existence implies that *ideal* outcomes are Markovian [34]. We now turn to imperfect measurements. Consider first the classical lumping. We set the shorthand notation $p_n^C[\{X_i, \tau_i\}] \equiv p_n^C[\{X_i, \tau_i\}_{i \in [0, n]}]$ such that

$$\begin{aligned} p_n^C[\{X_i, \tau_i\}] &= \left(\prod_{k=0}^n \sum_{x_k \in X_k} \right) p_n[\{x_i, \tau_i\}] \\ &= \left(\prod_{k=0}^n \sum_{x_k \in X_k} \right) T_{x_n}^{x_{n-1}} p_{n-1}[\{x_i, \tau_i\}], \end{aligned} \quad (5)$$

where we used Eq. (4). Note that if (and only if)

$$\sum_{x_k \in X_k} T_{x_k}^{x_{k-1}} = \sum_{x_k \in X_k} T_{x_k}^{y_{k-1}}, \quad \forall x_{k-1}, y_{k-1} \in X_{k-1}, \quad k \in \mathbb{N} \quad (6)$$

and any mesostate $X_{k-1} = A, B, \dots$, the classical lumping is Markovian with transition matrix $\bar{T}_X^Y \equiv \sum_{x \in X} T_x^{y \in Y}$

$$p_n^C[\{X_i, \tau_i\}] = \bar{T}_{X_n}^{X_{n-1}} p_{n-1}^C[\{X_i, \tau_i\}]. \quad (7)$$

Otherwise, the classical lumping is non-Markovian. The condition in Eq. (6) implies that any mesoscopic transition $A \rightarrow B$ for a given stroboscopic time τ is observed with equal probability, and it is thus impossible to store information in the history of mesostates analogously as for classical dynamics [32].

The stroboscopic quantum lumping is analogous to ideal measurements in Eq. (3) but with the high-rank projectors $\bar{\Pi}_X$ replacing Π_x , $\bar{\Pi}_X \circ \rho \equiv \bar{\Pi}_X \rho \bar{\Pi}_X$, yielding

$$\begin{aligned} p_n^Q[\{X_i, \tau_i\}] &= \\ &\text{Tr}\{\bar{\Pi}_{X_n} \circ \Lambda_\tau \circ \bar{\Pi}_{X_{n-1}} \circ \Lambda_\tau \circ \dots \circ \bar{\Pi}_{X_0} \circ \rho_0\}. \end{aligned} \quad (8)$$

Denoting by $\delta_{x,y}$ the Kronecker delta we can partition Eq. (8) as $p_n^Q[\{X_i, \tau_i\}] = p_n^C[\{X_i, \tau_i\}] + Q_n[\{X_i, \tau_i\}]$ with

$$Q_n[\{X_i, \tau_i\}] = \left(\prod_{k=0}^n \sum_{x_k, y_k \in X_k} \right) \left(1 - \prod_{k=0}^{n-1} \delta_{x_k, y_k} \right) \times \text{Tr}\{\Pi_{x_n} \Lambda_\tau \Pi_{x_{n-1}} \Lambda_\tau \cdots \rho_0 \cdots \Lambda_\tau^\dagger \Pi_{y_{n-1}} \Lambda_\tau^\dagger \Pi_{y_n}\}, \quad (9)$$

which is the *detectable coherence* that remains after the respective collapses in the Lüders measurement [9]. It depends on all the previously observed (meso)states of the system, including ρ_0 , so the joint probability distribution in Eq. (8) is generally *not* Markovian. The detectable coherence is a purely quantum memory effect having *no* classical analogue. If $Q_n = 0$ the classical and quantum lumped statistics coincide $p_n^Q[\{X_i, \tau_i\}] = p_n^C[\{X_i, \tau_i\}]$ and the coherence is either not generated in the unitary evolution or cannot be detected by the measurement.

Therefore, the classical lumped process is Markovian under the condition in Eq. (6), whereas quantum lumping additionally requires $Q_n = 0$ for all sequences $\{X_i, \tau_i\}_{i \in [0, n]}$ for the quantum-lumped process to be Markovian. If either Eq. (6) is violated or $Q_n \neq 0$, the observed process has memory. This is our main result.

Remark that, opposite to the von Neumann measurement, the Lüders measurement is independent on how we define the outcomes within the mesostates. In fact, any rotation of the outcomes basis which preserves the mesostates invariant, gives place to the same quantum lumping. The condition on Markovianity of the quantum lumping is independent on how we choose the outcome basis in Eqs. (6) and (9) (see Appendix).

4 An example

We illustrate the above results with an N -site quantum lattice (see Fig. 1d for $N = 4$) with states $|x\rangle$ and Hamiltonian

$$H = - \sum_{x=0}^{N-1} (|x+1\rangle \langle x| + |x\rangle \langle x+1|). \quad (10)$$

We consider $\hbar = 1$ and periodic boundary conditions. Without loss of generality N is assumed to be even. The eigenenergies are given by $\epsilon_k = -2 \cos(2\pi k/N)$ with corresponding eigenstates $\langle x | \epsilon_k \rangle = e^{-i2\pi kx/N} / \sqrt{N}$ for $k = 0, 1, \dots, N-1$

[70]. We assume a unitary evolution between consecutive measurements $\Lambda_\tau = U(\tau) = e^{-iH\tau}$. We first consider an ideal measurement detecting the system at site $|x\rangle$ and returning the outcome x . The transition matrix in Eq. (4) reads

$$\begin{aligned} T_x^y &= \left| \langle x | e^{-iH\tau} | y \rangle \right|^2 \\ &= \frac{1}{N^2} \sum_{k, l=0}^{N-1} e^{-i(\epsilon_k - \epsilon_l)\tau} e^{-i\frac{2\pi}{N}(k-l)(x-y)}, \end{aligned} \quad (11)$$

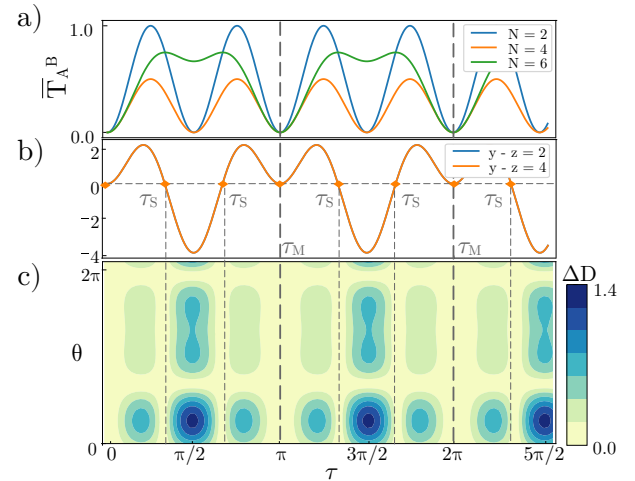


Figure 2: a) $\bar{T}_A^B$ as a function of τ for $N = 2, 4, 6$. b) Sum in Eq. (14) for $N = 6$ and $y - z = 2, 4$. The crossings with the horizontal line indicate the roots τ_M, τ_S . c) ΔD_2 for several values of τ and different initial preparations ρ_0, σ_0 parametrized by θ : $\rho_0 = |\psi_\theta\rangle \langle \psi_\theta|$ with $\sqrt{2}|\psi_\theta\rangle = |0\rangle + \cos\theta|2\rangle + \sin\theta|4\rangle$ and $\sigma_0 = |0\rangle \langle 0|/2 + |2\rangle \langle 2|/4 + |4\rangle \langle 4|/4$. The quantum lumped process is Markovian iff $\Delta D_2 \leq 0$.

In considering imperfect observations we define two mesostates $A = \{2j\}_{j=0}^{N/2-1}$ and $B = \{2j+1\}_{j=0}^{N/2-1}$ (see Fig. 1d for $N = 4$). This choice is based on real applications in quantum metrology [61] and computation [71]. In the last part of the paper, and also in Appendix B, we discuss that these mesostate configurations appear naturally in qubit chains under local measurements. We

have for any $N \geq 4$

$$\sum_{x \in A} T_x^y = \frac{1}{2} - \frac{1}{2N} \sum_{l=0}^{N-1} e^{-i4 \cos(\frac{2\pi l}{N})\tau} \equiv \bar{T}_A^B, \quad (12)$$

and since $\bar{T}_A^B$ does not depend on $y \in B$ the condition (6) is satisfied and p_n^C is Markovian for any τ .

In contrast, quantum lumping is generally non-Markovian because of the additional non-zero term Q_n ; for simplicity we here consider only $n = 1$, the result for any n is given in Appendix A. According to Eq. (8) we find, setting $\rho_0^{yz} \equiv \langle y | U \rho_0 U^\dagger | z \rangle$,

$$Q_1[A, \tau; B, 0] = \frac{1}{2N} \times \sum_{z, y \neq z \in B} \sum_{l=0}^{N-1} \left(1 - e^{-i4 \cos(\frac{2\pi l}{N})\tau}\right) e^{i\frac{2\pi l}{N}l(y-z)} \rho_0^{yz}. \quad (13)$$

Q_1 is zero for $A \rightarrow B$ and $B \rightarrow A$ if (i) the initial state contains no coherence, $\rho_0^{yz} = 0$, (ii) if $e^{-i4 \cos(2\pi l \tau/N)} = 1$, or (iii) if the sum over l in Eq. (13) vanishes. (i) depends on the particular preparation of the system, while (iii) corresponds to values $\tau = \tau_S$ at which

$$\sum_{l=0}^{N-1} \left(1 - e^{-i4 \cos(\frac{2\pi l}{N})\tau_S}\right) e^{i\frac{4\pi l}{N}l(y-z)} = 0, \quad \forall z, y \neq z \in B. \quad (14)$$

At stroboscopic times τ_S , the quantum and classical lumped distributions coincide, $p_n^Q = p_n^C$ and are both Markovian as they evolve with time. In Fig. 2b we show the sum (14) for $y - z = 2, 4$ and $N = 6$ to confirm our prediction. In this particular case the curves coincide and we have $\tau_S = s\pi/3$.

The case (ii) corresponds to the “magic” times τ_M satisfying $4 \cos(2\pi l \tau_M/N) = \pm s2\pi$ for $s \in \mathbb{N}_0$ and $l = 0, \dots, N-1$. At these times, the transition matrix $\bar{T}_A^B$ vanishes. Fig. 2a depicts $\bar{T}_A^B$ for several values of τ ; the “magic” times are $\tau_M = s\pi/2$ for $N = 4$, $\tau_M = s\pi$ for $N = 6$, whereas for $N = 8$ we find $\tau_M = 0$ as the unique solution. Stroboscopic measurements with τ_M are *not* able to detect any dynamics between the mesostates even if transitions do in fact occur. A similar phenomenon was observed in first-detection problems [5–7].

We now confirm the presence of purely quantum memory. We evaluate the marginal distributions of the n -th outcome, $P_n^Q[X] \equiv$

$(\prod_{k=0}^{n-1} \sum_{X_k}) p_n^Q[\{X_i, (i-1)\tau\}]$ with two different initial preparations ρ_0 and σ_0 parametrized by an angle θ (see caption to Fig. 2). The Kolmogorov distance between them is $D_n = \sum_X |P_{n-1}^Q[X](\rho_0) - P_{n-1}^Q[X](\sigma_0)|$. If the lumped distributions are Markovian, the trace distance is nonincreasing between consecutive measurements, $\Delta D_n = D_{n+1} - D_n \leq 0$ for any $n \in \mathbb{N}$; otherwise the evolution is non-Markovian [36]. Fig. 2c shows $\Delta D_1 = D_2 - D_1$ for different values of τ and different ρ_0 and σ_0 . The quantum-lumped process is non-Markovian in the blue regions where $\Delta D_1 > 0$ and vanishes at τ_S and τ_M in agreement with our prediction. As expected, the memory also depends on initial correlations, with unobservable coherences appearing in two horizontal bands with $\Delta D_1 = 0$ at two different values of θ , independent of τ .

5 Discussion

We analyzed the non-Markovian behavior resulting from imperfect stroboscopic quantum measurements. The von Neumann imperfect measurement produces a mixed quantum state averaged over all lumped (i.e. degenerate) outcomes. Conversely, the Lüders measurement results in a multi-dimensional projection preserving some quantum coherence. In both cases, the statistics of the indistinguishable measurement outcomes are coarse-grained, lumped distributions, which evolve in time in a generally non-Markovian fashion. The presence and characteristics of memory were shown to depend tacitly on the measurement, in particular on the nature of, and projection onto, mesostates. As a main result, the quantum-lumped process requires stronger conditions to be Markovian because of preserved quantum coherence. In particular, it requires the observation to have *no* detectable coherence. In turn, we introduced the concept of *purely quantum* memory, which has *no classical counterpart*. It corresponds to the situation where the respective mesoscopic transition for a given stroboscopic time τ are observed with equal probability (i.e. are Markovian with respect to von Neumann lumping) and the memory emerges purely due to detectable coherence. We illustrated these results by means of a quantum walk on a lattice and determined the parameters for which the respective lumped distributions are Markovian. The condi-

tion Eq.(6), as well as the magic times τ_M , are very sensitive to the symmetry of the quantum lattice and the mesostates choice.

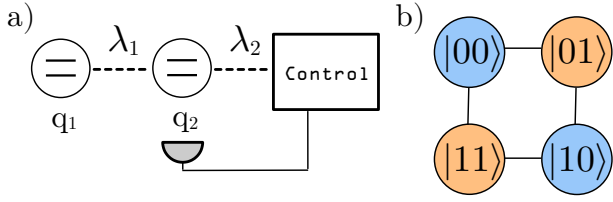


Figure 3: Schematic of the qubit chain setup: a) schematic of the interactions between the qubits and the external control. Measurements are performed on q_2 exclusively. b) Possible transitions between the two qubits states created by arbitrary couplings λ_1 and λ_2 . Colors denote the respective indistinguishable states under measurements of q_2 , analogous to those studied in the quantum lattice with $N = 4$.

As mentioned, the Lüders measurement is noise free whereas the von Neumann version applies in the presence of strong decoherence, where we note that the relationship between the stroboscopic measurement time τ and the decoherence timescale may be very relevant in practice. Future work should extend our results to more realistic measurement models, explicitly introducing dissipative (Lindblad) dynamics. A simple, but conceptually interesting design of this kind is sketched in Fig. 3a. The device consists in two coupled qubits q_1 and q_2 . An external observer measures only q_2 . In Appendix B we show that, in the absence of noise, these measurements behave as Lüders measurements because of the coherence remaining in q_1 . In the presence of a strong decoherence mechanism, this coherence is removed and von Neumann measurements are recovered. Weak decoherence mechanisms are in turn expected to define intermediate types of lumping. The lumpings created by this setup are equivalent to that studied in the article, with $N = 4$, see Fig. 3b.

This connects our results to other works such as Ref. [48], where non-Markovianity was analyzed from the perspective of the distinction between a quantum and a classical environment. In fact, Fig. 3 shows how a Lüders imperfect measurement becomes analog to an ideal measurement on an smaller system (qubit 2) entangled with an environment (qubit 1). This analogy will depend on the mesostate configuration and the Hamiltonian connecting them. In [48] the authors provide

a method to discern between classical and quantum memory, employing entanglement-detection techniques. The consecutive application of von-Neumann and Lüders measurements in turn also provides a simple method to discern quantum coherence, as shown by our example. These two tests are in fact complementary and can both be used in a laboratory, depending on the features and capabilities of the experimental device.

The above discussion relates to the ongoing debate about how Markovian behaviour is recovered in quantum systems from pure dynamics (see, for example, Refs. [72–75]). In fact, given a multipartite quantum system under local or imperfect measurements, the recovered statistics happen to be non-Markovian. The aforementioned works show that many-body quantum systems recover Markovianity under certain approximations. In the present work we derived *exact* conditions required to recover Markovian behavior, where both classical and quantum memory are absent, independent of the size of the system. However, we observe that the respective conditions in practice require extremely symmetric Hamiltonians and mesostate configurations. This sensitivity of memory effects in small systems can be exploited to improve the performance of quantum devices, where Markovianity plays an important role, for example, in quantum thermodynamics (see discussion in [76]), quantum communication [77], and control [78]. However, one of the main motivations for the present work was to establish a first connection between properties of quantum measurements and certain important ongoing debates in the non-equilibrium thermodynamics of classical systems [18–31]. These works highlight that lumped measurements play a central role in defining and inferring dissipation (i.e. the thermodynamic entropy production) in coarsely observed systems out of equilibrium. To the best of our knowledge this debate remains unexplored from a quantum perspective.

Note that the effect of measurements on (de)coherence in open quantum systems is highly non-trivial. For example, creating coherence during Landauer’s erasure implies additional energy dissipation [79]. Conversely, it is possible to extract work from coherences during a quantum measurements [80, 81]. Other works revealed the implications of measurement-induced decoherence in entropy generation [82] and re-

covering thermalization [83]. Our work motivates generalizations of the above results to imperfect measurements and provokes fundamental questions about the implications of quantum/classical lumping for quantum thermodynamics. An example is the counting statistics of systems under continuous monitoring, such as electric transport along quantum dots in contact with electric Fermionic leads or photoelectric detection [84–88]. In these experiments the environment (the electromagnetic field or the Fermionic leads) ‘performs’ the measurements, because small changes in the quantum system can be directly detected as single-photon emission, or single-electron transport. This typically corresponds to a von Neumann lumping, since the only source of noise in the results is generically measurement noise. However, the example in Fig. 3 teaches us that the von-Neumann measurement requires a strong dephasing during the measurement process. Since the control and isolation of quantum technologies are improving fast, Lüders measurements will become relevant in the near future, and our formalism could find important applications in these kinds of experiments.

Finally, while we focused on unitary evolution between consecutive measurements, our results apply to more complex inter-measurement evolutions, provided that Eqs. (3) and (8) hold. This is *not* always the case in open quantum systems, where the contact with an environment may lead to violations of the quantum regression formula [2, 34]. The relationship between non-Markovianity, the quantum regression formula, and the quantum measurement has recently been studied extensively [10, 40, 89]. The lumping described in this work introduces an additional level of complexity; the role of the contact with the environment on the measurement statistics yet remains unexplored.

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A Details on the quantum lattice

Here we derive equations Eqs. (12) and (13) in the main text. We start by summing over $x \in A$ in Eq. (11),

$$\sum_{x \in A} T_x^y = \frac{1}{N^2} \sum_{k,l} e^{-i(\epsilon_k - \epsilon_l)\tau} e^{i\frac{2\pi}{N}(k-l)y} \sum_{x \in A} e^{-i\frac{2\pi}{N}(k-l)x}. \quad (15)$$

This sum depends explicitly on y ; somewhat surprisingly this dependence disappears if we restrict x and y to the chosen mesostates. Observe that the last sum

$$\sum_{x \in A} e^{-i\frac{2\pi}{N}(k-l)x} = \sum_{j=0}^{N/2-1} e^{-i\frac{4\pi}{N}(k-l)j}, \quad (16)$$

is a geometric series that we can evaluate to find

$$\sum_{j=0}^{N/2-1} e^{-i\frac{4\pi}{N}(k-l)j} = \begin{cases} N/2, & \text{if } k-l = \pm sN/2 \\ \frac{1-e^{-i2\pi(k-l)}}{1-e^{-i\frac{4\pi}{N}(k-l)}}, & \text{otherwise} \end{cases}, \quad (17)$$

where $s = 0, 1, 2, \dots$ is a non-negative integer, k and l are also integers restricted between 0 and $N-1$. Since $k-l$ is an integer the lower case always gives a null contribution to the series and we must only consider $k-l = \pm sN/2$. In addition, the equations $k-l = \pm sN/2$ have solutions only for $s = 0, 1$: for $s = 0$ the solution is $k = l = 0, 1, \dots, N-1$ while for $s = 1$ we have $k = N/2 + l$ with $l = 0, 1, \dots, N/2$ and $k = l - N/2$ with $l = N/2, \dots, N-1$. Inserting this into Eq. (15) we obtain

$$\begin{aligned} \sum_{x \in A} T_x^y &= \frac{1}{2N} \sum_{k-l=\pm sN/2} e^{-i(\epsilon_k - \epsilon_l)\tau} e^{\pm i\pi sy} \\ &= \frac{1}{2} + \frac{1}{2N} e^{\pm i\pi y} \sum_{k-l=\pm N/2} e^{-i(\epsilon_k - \epsilon_l)\tau}. \end{aligned} \quad (18)$$

Since $y \in B$ are an odd integers, $e^{\pm i\pi y} = -1$ for all B , so

$$\bar{T}_A^B = \frac{1}{2} - \frac{1}{2N} \sum_{k-l=\pm N/2} e^{-i(\epsilon_k - \epsilon_l)\tau}, \quad (19)$$

which satisfies Eq. (6). Therefore, the classical lumping process in this particular example is Markovian. To obtain more insight about the dependence of $\bar{T}_A^B$ on τ we consider the eigenenergies explicitly,

$$\begin{aligned} \epsilon_k - \epsilon_l &= -2 \cos \frac{2\pi k}{N} + 2 \cos \frac{2\pi l}{N} \\ &= -2 \cos \left[\frac{2\pi}{N}(l \pm N/2) \right] + 2 \cos \frac{2\pi l}{N}. \end{aligned} \quad (20)$$

Using the identity $\cos \left[\frac{2\pi}{N}(l \pm N/2) \right] = -\cos \frac{2\pi l}{N}$ we have

$$\epsilon_k - \epsilon_l = 4 \cos \frac{2\pi l}{N}, \quad (21)$$

from which we obtain Eq. (12) in the main text.

To determine the detectable coherence, we use the spectral expansion of the Hamiltonian H in Eq. (10) to obtain

$$\begin{aligned} Q_1[A, \tau; B, 0] &= \frac{1}{N^2} \sum_{z, y \neq z \in B} \\ &\sum_{k,l} e^{-i(\epsilon_k - \epsilon_l)\tau + i\frac{2\pi}{N}(ky - lz)} \sum_{x \in A} e^{-i\frac{2\pi}{N}(k-l)x} \rho_0^{yz}. \end{aligned} \quad (22)$$

The sum over $x \in A$ is similar to that in Eq. (15), so we consider the restriction $k - l = \pm sN/2$, with $s = 0, 1$,

$$\begin{aligned}
Q_1[A, \tau; B, 0] &= \\
&= \frac{1}{2N} \sum_{z, y \neq z \in B} \sum_{k-l=\pm sn} e^{-i(\epsilon_k - \epsilon_l)\tau} e^{i\frac{2\pi}{N}(ky-lz)} \rho_0^{yz} \\
&= \frac{1}{2N} \sum_{z, y \neq z \in B} \sum_{l=0}^{N-1} e^{i\frac{2\pi}{N}l(y-z)} \rho_0^{yz} \\
&\quad + \frac{1}{2N} \sum_{z, y \neq z \in B} \sum_{l=0}^{N-1} e^{-i4\cos(\frac{2\pi l}{N})\tau} e^{i\frac{2\pi}{N}[l(y-z) \pm \frac{N}{2}y]} \rho_0^{yz} \\
&= \frac{1}{2N} \sum_{z, y \neq z \in B} \sum_{l=0}^{N-1} \left(1 - e^{-i4\cos(\frac{2\pi l}{N})\tau}\right) e^{i\frac{2\pi}{N}l(y-z)} \rho_0^{yz},
\end{aligned} \tag{23}$$

which is Eq. (13) in the main text.

Q_n for any n follows by noticing that $\text{Tr}\{\Pi_{x_n} \cdots \Pi_{y_n}\} = \text{Tr}\{\Pi_{x_n} \cdots \Pi_{x_n}\} \delta_{x_n, y_n}$ by the cyclic property, writing the trace in Eq. (9) as

$$\begin{aligned}
&\text{Tr}\{\Pi_{x_n} U \Pi_{x_{n-1}} U \cdots \rho_0 \cdots U^\dagger \Pi_{y_{n-1}} U^\dagger \Pi_{y_n}\} = \\
&\delta_{x_n, y_n} \langle x_0 | \rho_0 | y_0 \rangle \prod_{i=1}^n \langle x_i | U | x_{i-1} \rangle \langle y_{i-1} | U^\dagger | y_i \rangle,
\end{aligned} \tag{24}$$

and inspecting the $k = n$ term in Eq. (9) as

$$\begin{aligned}
&\sum_{x_n \in X_n} \langle x_n | U | x_{n-1} \rangle \langle x_n | U | x_{n-1} \rangle \langle y_{n-1} | U^\dagger | x_n \rangle \langle y_{n-1} | U^\dagger | x_n \rangle = \frac{1}{N^2} \times \\
&\sum_{k, l} e^{-i(\epsilon_k - \epsilon_l)\tau + i\frac{2\pi}{N}(ky_{n-1} - lx_{n-1})} \sum_{x_n \in X_n} e^{-i\frac{2\pi}{N}(k-l)x_n}.
\end{aligned} \tag{25}$$

The above sum is analogous to that in Eq. (22), and since it enters multiplicatively into Eq. (9) we conclude that Q_n vanish for the same "magic" times τ_M and τ_S in Eq. (14) for any n .

B Two-qubits example

In most practical applications imperfect measurements occur when we measure only a controllable part but not a complete quantum system [34]. The measured portion is known as the *quantum probe*. The rest of the system remains unaffected by the measurement, thus hiding part of the information and in turn giving rise the Lüders and von Neumann schemes, as we show. We now consider an explicit example of a quantum imperfect measurement to gain insight into the respective measurement schemes that we postulated in the main text. The example also establishes a link between the example in the main text and applications in quantum technologies.

We consider a system composed by two qubits, q_1 and q_2 , see Fig. 3. Qubit q_2 plays the role of the quantum probe and can be measured and manipulated externally. q_1 is the hidden part of the system interacting only with q_2 . We consider the Hamiltonian $H = \lambda_1 \sigma_x \otimes \sigma_x + \lambda_2 \mathbb{I} \otimes \sigma_x$. The first term represents the qubit-qubit interaction and the second one an external driving, controlling qubit q_2 , which represents the quantum probe in this case. λ_1 and λ_2 are the intensity of the couplings. Note that with this choice of Hamiltonian, this system is equivalent to the quantum lattice in the main text (Eq. (10)) with $N = 4$ states denoted by $|0\rangle = |00\rangle, |1\rangle = |01\rangle, |2\rangle = |10\rangle$ and $|3\rangle = |11\rangle$. This boundary-driven qubit-chain model is in fact a paradigmatic example in quantum computing [90]. In recent years it has found even more applications, for example, in quantum transport [91, 92] or

quantum thermodynamics [93, 94]. The local single-qubit measurements deeply affect the performance of both, thermal machines and transport systems [91, 95].

If the initial state of the joint system is pure and the evolution remains unitary, at an arbitrary measurement time t the state will take the form

$$|\psi_{12}\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{10} |10\rangle + \alpha_{11} |11\rangle, \quad (26)$$

where α_{ij} are time-dependent coefficients. If we measure q_2 on this state and obtain the outcome $q_2 = 0$, the resulting state after the measurement is

$$(\mathbb{I} \otimes |0\rangle\langle 0|) |\psi_{12}\rangle = (\alpha_{00} |0\rangle + \alpha_{10} |1\rangle) \otimes |0\rangle, \quad (27)$$

up to normalization. If we measure $q_2 = 1$, the resulting state is

$$(\mathbb{I} \otimes |1\rangle\langle 1|) |\psi_{12}\rangle = (\alpha_{01} |0\rangle + \alpha_{11} |1\rangle) \otimes |1\rangle. \quad (28)$$

Observe that this operation is equivalent to the Lüders measurement with four outcomes $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ lumped into two indistinguishable mesostates, $A = \{|00\rangle, |10\rangle\}$ and $B = \{|01\rangle, |11\rangle\}$. In this case, measuring q_2 only determines the mesostate and *not* the microscopic state of the system. The associated projectors are precisely $\Pi_A = |00\rangle\langle 00| + |10\rangle\langle 10| = \mathbb{I} \otimes |0\rangle\langle 0|$ and $\Pi_B = |11\rangle\langle 11| + |01\rangle\langle 01| = \mathbb{I} \otimes |1\rangle\langle 1|$.

The situation is different if the evolution is dissipative. In this case, the system could relax into a diagonal mixed state

$$\begin{aligned} \rho_{12} = & r_{00} |00\rangle\langle 00| + r_{01} |01\rangle\langle 01| \\ & + r_{10} |10\rangle\langle 10| + r_{11} |11\rangle\langle 11|. \end{aligned} \quad (29)$$

Here r_{ij} are coefficients obeying $r_{00} + r_{10} + r_{01} + r_{11} = 1$. If we measure q_2 and obtain the outcome $q_2 = 0$, the resulting diagonal density matrix is

$$r_{00} |00\rangle\langle 00| + r_{10} |10\rangle\langle 10|, \quad (30)$$

up to normalization, and

$$r_{11} |11\rangle\langle 11| + r_{01} |01\rangle\langle 01| \quad (31)$$

if we measure $q_2 = 1$. The resulting matrices are again lumped into the same mesostates A and B . However, opposite to the pure case above, the quantum coherence is removed from the system, corresponding to the von Neumann measurement scheme.

C General quantum Markov processes

In the main text, we determine the conditions required by the system in order to recover Markovian behaviour, given a measurement basis which define the von Neumann-Lüders schemes. To complete our study, in this appendix we derive these conditions in a basis-independent way. Our starting point is the definition of a quantum Markovian lumping,

$$p_n^Q[\{X_i, \tau_i\}] = \bar{T}_{X_n}^{X_{n-1}} p_{n-1}^Q[\{X_i, \tau_i\}] \quad (32)$$

where $\bar{T}_{X_n}^{X_{n-1}}$ is a transition matrix between mesostates. Substituting Eq.(8) into this definition we have

$$\text{Tr}\left\{\bar{\Pi}_{X_n} U \bar{\Pi}_{X_{n-1}} \rho_{n-1} \bar{\Pi}_{X_{n-1}} U^\dagger\right\} = \bar{T}_{X_n}^{X_{n-1}} \text{Tr}\left\{\bar{\Pi}_{X_{n-1}} \rho_{n-1}\right\} \quad (33)$$

ρ_{n-1} represents the state of the system before the $n - 1$ measurement, which contains information about the initial preparation of the system and the previous measurements. We can write this in a more compact way defining the operators $\Phi_{X_n}^{X_{n-1}} = \bar{\Pi}_{X_{n-1}} U^\dagger \bar{\Pi}_{X_n} U \bar{\Pi}_{X_{n-1}}$, then

$$\text{Tr}\left\{\Phi_{X_n}^{X_{n-1}} \rho_{n-1} \bar{\Pi}_{X_{n-1}}\right\} = \bar{T}_{X_n}^{X_{n-1}} \text{Tr}\left\{\rho_{n-1} \bar{\Pi}_{X_{n-1}}\right\}. \quad (34)$$

Where we used the cyclic property of the trace. The eigenbasis of each operator $\Phi_{X_n}^{X_{n-1}}$ is defined by $\Phi_{X_n}^{X_{n-1}} |\lambda_{X_n}^{X_{n-1}}\rangle = \lambda_{X_n}^{X_{n-1}} |\lambda_{X_n}^{X_{n-1}}\rangle$, $\lambda = 1, 2, \dots, \text{rank}(\bar{\Pi}_{X_{n-1}})$. Expanding the trace on this basis we have

$$\begin{aligned} \sum_{\lambda} \lambda_{X_n}^{X_{n-1}} \langle \lambda_{X_n}^{X_{n-1}} | \rho_{n-1} \bar{\Pi}_{X_{n-1}} | \lambda_{X_n}^{X_{n-1}} \rangle &= \\ = \bar{T}_{X_n}^{X_{n-1}} \sum_{\lambda} \langle \lambda_{X_n}^{X_{n-1}} | \rho_{n-1} \bar{\Pi}_{X_{n-1}} | \lambda_{X_n}^{X_{n-1}} \rangle. \end{aligned} \quad (35)$$

Remark that, in general, one could find solutions to this equation for particular configurations of ρ_0 , U and mesostate configurations. In these particular cases, the evolution of the quantum lumping is Markovian. However, we can find solutions of the last equation independent of ρ_0 , which are therefore Markovian for any initial condition. In these solutions, the terms inside the sums in the last equation must cancel one by one,

$$\begin{aligned} \lambda_{X_n}^{X_{n-1}} \langle \lambda_{X_n}^{X_{n-1}} | \rho_{n-1} \bar{\Pi}_{X_{n-1}} | \lambda_{X_n}^{X_{n-1}} \rangle &= \\ = \bar{T}_{X_n}^{X_{n-1}} \langle \lambda_{X_n}^{X_{n-1}} | \rho_{n-1} \bar{\Pi}_{X_{n-1}} | \lambda_{X_n}^{X_{n-1}} \rangle. \end{aligned} \quad (36)$$

for any λ . This means the operator $\Phi_{X_n}^{X_{n-1}}$ is fully degenerated, and its unique eigenvalue is, precisely $\bar{T}_{X_n}^{X_{n-1}}$.

To recapitulate, if the quantum lumping is Markovian for any initial preparation, then $\Phi_{X_n}^{X_{n-1}}$ (which only depends on the unitary evolution and the mesostates choice, but not in the outcome basis) is fully degenerated, and the transition $\bar{T}_{X_n}^{X_{n-1}}$ matrix coincides with its unique eigenvalue. The last must be true for all the mesostates X_n .

Given two degenerated eigenstates $|\lambda_{X_n}^{X_{n-1}}\rangle$ and $|\mu_{X_n}^{X_{n-1}}\rangle$, with $\lambda \neq \mu$, we have the identity $\langle \lambda_{X_n}^{X_{n-1}} | \Phi_{X_n}^{X_{n-1}} | \lambda_{X_n}^{X_{n-1}} \rangle = \langle \mu_{X_n}^{X_{n-1}} | \Phi_{X_n}^{X_{n-1}} | \mu_{X_n}^{X_{n-1}} \rangle$. Expanding the operator we have

$$\begin{aligned} \langle \lambda_{X_n}^{X_{n-1}} | U^\dagger \bar{\Pi}_{X_n} U | \lambda_{X_n}^{X_{n-1}} \rangle &= \sum_{x_n \in X_n} |\langle x_k | U | \lambda_{X_n}^{X_{n-1}} \rangle|^2 \\ = \sum_{x_n \in X_n} |\langle x_k | U | \mu_{X_n}^{X_{n-1}} \rangle|^2 &= \langle \mu_{X_n}^{X_{n-1}} | U^\dagger \bar{\Pi}_{X_n} U | \mu_{X_n}^{X_{n-1}} \rangle \end{aligned} \quad (37)$$

Here $|x_n\rangle$ is a basis of the mesostate X_n as in the main text. The former proves that, if the quantum lumping is Markovian for any initial preparation, Eq.(6) is satisfied. We notice that if $\Phi_{X_n}^{X_{n-1}}$ is degenerated, the basis $|\lambda_{X_n}^{X_{n-1}}\rangle$ is not uniquely determined. In fact, any rotation of $|\lambda_{X_n}^{X_{n-1}}\rangle$ preserving the mesostate structure still satisfies the equations in this section, each one defining a particular von Neumann measurement.

Expanding the projectors $\bar{\Pi}_{X_{n-1}} = \sum_{\lambda} |\lambda_{X_n}^{X_{n-1}}\rangle \langle \lambda_{X_n}^{X_{n-1}}| \equiv \sum_{\lambda} \bar{\Pi}_{\lambda}$ and substituting in Eq.(8) we see that

$$p_n^Q[\{X_i, \tau_i\}] = \sum_{\lambda} \text{tr} \left\{ \bar{\Pi}_{X_n} \circ U \circ \bar{\Pi}_{\lambda} \circ \dots \circ \rho_0 \right\}. \quad (38)$$

As we can see, in the basis $|\lambda_{X_n}\rangle$, the quantum lumping coincides with the classical one defined in Eq.(7), so detectable coherence is zero.